

```
ics
(depth < MAXDEPTH)
inside ? 1 : 0;
nt = nt / nc; ddn = abs(
s2t = 1.0f - nnt * nnt;
D, N );
)
at a = nt - nc; b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) *
Tr) R = (D * nnt - N * (ddn
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse,
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, &light;
e.x + radiance.y + radiance.z) > 0) && (abs(
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following
ive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```

/INFOMOV/

# Optimization & Vectorization

J. Bikker - April-June 2024 - Lecture 2: "Low Level"

# Welcome!



# Today's Agenda:

- The Cost of a Line of Code
- CPU Architecture: Instruction Pipeline
- Data Types and Their Cost
- Rules of Engagement



# Instruction Cost

What is the ‘cost’ of a multiply?

```
starttimer();
float x = 0;
for( int i = 0; i < 1000000; i++ ) x *= y;
stoptimer();
```

- Actual measured operations:
  - timer operations;
  - initializing ‘x’ and ‘i’;
  - comparing ‘i’ to 1000000 (x 1000000);
  - increasing ‘i’ (x 1000000);
  - jump instruction to start of loop (x 1000000).
- Compiler outsmarts us!
  - No work at all unless we use x
  - $x += 1000000 * y$

**Better solution:**

- Create an arbitrary loop
- Measure time with and without the instruction we want to time

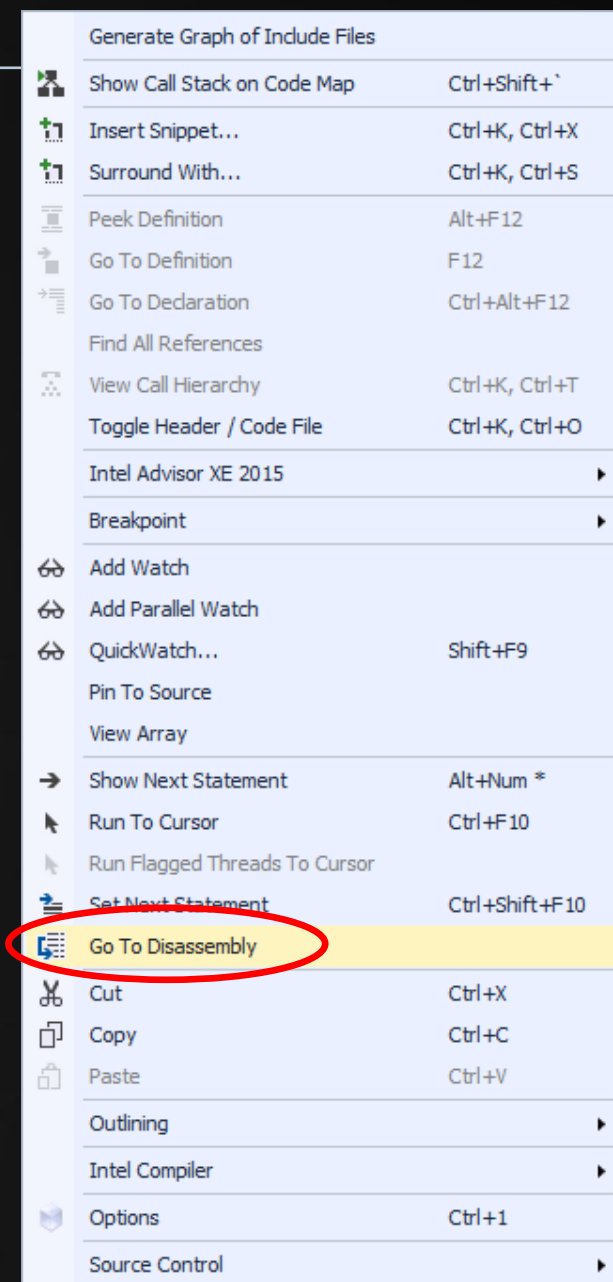


# Instruction Cost

What is the ‘cost’ of a multiply?

```

float x = 1, y = 1;
unsigned int i = 0, j = 0x28929227;
for( int k = 0; k < ITERATIONS; k++ )
{
    // ensure we feed our line with fresh data
    x += y, y *= 0.9999f;
    // integer operations to free up fp execution units
    i += j, j ^= 0x17737352, i >>= 1, j /= 28763;
    // operation to be timed
    if (with) x *= y;
    // integer operations to free up fp execution units
    i += j, j ^= 0x17737352, i >>= 1, j /= 28763;
}
dummy = x + (float)i;
    
```



# Instruction Cost

x86 assembly in 5 minutes

Modern CPUs still run x86 machine code, based on Intel’s 1978 8086 processor. The original processor was 16-bit, and had 8 ‘general purpose’ 16-bit registers\*:

AX (‘accumulator register’)	AH, AL (8-bit)	EAX (32-bit)	RAX (64-bit)
BX (‘base register’)	BH, BL	EBX	RBX
CX (‘counter register’)	CH, CL	ECX	RCX
DX (‘data register’)	DH, DL	EDX	RDX
BP (‘base pointer’)		EBP	RBP
SI (‘source index’)		ESI	RSI
DI (‘destination index’)		EDI	RDI
SP (‘stack pointer’)		ESP	RSP
		st0..st7	R8..R15
		XMM0..XMM7	XMM0..XMM15
			YMM0..YMM15
			ZMM0..ZMM31

\* More info: <http://www.swansontec.com/sregisters.html>



# Instruction Cost

x86 assembly in 5 minutes:

Typical assembler:

loop:

```

mov eax, [0x1008FFA0] // read from address into register
shr eax, 5           // shift eax 5 bits to the right
add eax, edx         // add registers, store in eax
dec ecx             // decrement ecx
jnz loop            // jump if not zero
fld [esi]           // load from address [esi] onto FPU
fld st0             // duplicate top float
faddp               // add top two values, push result

```

More on x86 assembler: <http://www.cs.virginia.edu/~evans/cs216/guides/x86.html>

A bit more on floating point assembler: [https://www.cs.uaf.edu/2007/fall/cs301/lecture/11\\_12\\_floating\\_asm.html](https://www.cs.uaf.edu/2007/fall/cs301/lecture/11_12_floating_asm.html)





# Instruction Cost

What is the ‘cost’ of a multiply?

```

float x = 0, y = 0.1f;
unsigned int i = 0, j = 0x28929227;
for( int k = 0; k < ITERATIONS; k++ )
{
    // ...
    x += y, y *= 1.01f;
    // ...
    i += j, j ^= 0x17737352, i >>= 1, j /= 28763;
    // ...
    if (with) x *= y;
    i += j, j ^= 0x17737352, i >>= 1, j /= 28763;
}
dummy = x + (float)i;
    
```

```

fldz
xor ecx, ecx
fld dword ptr ds:[405290h]
mov edx, 28929227h
fld dword ptr ds:[40528Ch]
push esi
mov esi, 0C350h = 50000
    
```

```

add ecx, edx
mov eax, 91D2A969h
xor edx, 17737352h
shr ecx, 1
mul eax, edx
fld st(1)
faddp st(3), st
    
```

$$= \frac{2^{46}}{28763} (!!)$$

```

mov eax, 91D2A969h
shr edx, 0Eh
add ecx, edx
fmul st(1),st
xor edx, 17737352h
shr ecx, 1
mul eax, edx
shr edx, 0Fh
dec esi
jne tobetimed<0>+1Fh
    
```



# Instruction Cost

What is the ‘cost’ of a multiply?

Observations:

- Compiler reorganizes code
- Compiler cleverly evades division
- Loop counter *decreases*
- Presence of integer instructions affects timing  
*(to the point where the mul is free)*

But also:

- It is really hard to measure the cost of a line of code.

```

ics
& (depth < MAXDEPTH)
{
    int n = inside ? 1 : 0;
    float nt = nt / nc; ddn = ddn * nc;
    float cos2t = 1.0f - nnt * nnt;
    Vec D, N );
    Vec R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse, r1, r2, &R, Spdf );
    estimation - doing it properly, closely following Section 2.1.2
    if;
    radiance = SampleLight( &rand, I, &L, &align );
    Vec e.x + radiance.y + radiance.z) > 0) && (rand < survive)
    w = true;
    Vec brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    float3 factor = diffuse * INVPI;
    Vec weight = Mis2( directPdf, brdfPdf );
    float cosThetaOut = dot( N, L );
    Vec E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Section 2.1.2
    survive)
    ;
    Vec3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, Spdf );
    survive;
    pdf;
    Vec n = E * brdf * (dot( N, R ) / pdf);
    mission = true;
    
```





# Instruction Cost

What is the ‘cost’ of a single instruction?

Cost is highly dependent on the surrounding instructions, and many other factors. However, there is a ‘cost ranking’:

- << >> *bit shifts*
- + - & | ^ *simple arithmetic, logical operands*
- \* *multiplication*
- / *division*
- sqrt
- sin, cos, tan, pow, exp

This ranking is generally true for any processor (including GPUs).

```

ics
& (depth < MAXDEPTH)
c = inside ? 1.0f : 0.0f;
nt = nt / nc; ddn = ddn * ddn;
os2t = 1.0f - nnt * nnt;
D, N );
)
at a = nt - nc, b = nt * nc;
at Tr = 1 - (R0 + (1 - R0) * c);
Tr) R = (D * nnt - N * (ddn *
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse, r);
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, Align
e.x + radiance.y + radiance.z) > 0) && (radiance
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following Seeley's
vive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, Spdf
urvive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```



# Instruction Cost

AMD K7  
1999

Instruction	Operands	Ops	Latency	Reciprocal throughput	Execution unit	Notes
<b>Arithmetic instructions</b>						
ADD, SUB	r,r/i	1	1	1/3	ALU	
ADD, SUB	r,m	1	1	1/2	ALU, AGU	
ADD, SUB	m,r	1	7	2,5	ALU, AGU	
ADC, SBB	r,r/i	1	1	1/3	ALU	
ADC, SBB	r,m	1	1	1/2	ALU, AGU	
ADC, SBB						
CMP						
CMP						
INC, DEC, NEG						
INC, DEC, NEG						
AAA, AAS						
DAA						
DAS						
AAD						
AAM						
MUL, IMUL						
MUL, IMUL	r16/m16	3	3	2	ALU0_1	latency ax=3, dx=4
MUL, IMUL	r32/m32	3	4	3	ALU0_1	
IMUL	r16,r16/m16	2	3	2	ALU0	
IMUL	r32,r32/m32	2	4	2,5	ALU0	
IMUL	r16,(r16),i	2	4	1	ALU0	
IMUL	r32,(r32),i	2	5	2	ALU0	
IMUL	r16,m16,i	3		2	ALU0	
IMUL	r32,m32,i	3		2	ALU0	
DIV	r8/m8	32	24	23	ALU	
DIV	r16/m16	47	24	23	ALU	
DIV	r32/m32	79	40	40	ALU	
IDIV	r8	41	17	17	ALU	
IDIV	r16	56	25	25	ALU	
IDIV	r32	88	41	41	ALU	
IDIV	m8	42	17	17	ALU	
IDIV	m16	57	25	25	ALU	
IDIV	m32	89	41	41	ALU	
<b>Math</b>						
FSQRT				1	35	12
FSIN				44	90-100	
FCOS				51	90-100	
FSINCOS				76	100-150	
FPTAN				46	100-200	
FPATAN				72	160-170	
FSCALE				5	8	
FEXTRACT				7	11	
F2XM1				8	27	
FYL2X				49	126	
FYL2XP1				63	147	



# Instruction Cost

## AMD Jaguar 2013

Instruction	Operands	Ops	Latency	Reciprocal throughput	Execution pipe	Notes
<b>Arithmetic instructions</b>						
ADD, SUB	r,r/i	1	1	0.5	10/1	
ADD, SUB	r,m	1		1		
ADD, SUB	m,r	1	6	1		
ADC, SRR	r,r/i	1	1	1	10/1	
ADC, SRR	<b>Math</b>					
ADC, SRR	FSQRT		1	35	35	FP1
CMP	FLDPI, etc.		1		1	FP0
CMP	FSIN		4-44	30-139	30-151	FP0, FP1
INC, D	FCOS		11-51	38-93		FP0, FP1
INC, D	FSINCOS		11-76	55-122	55-180	FP0, FP1
AAA	FPTAN		11-45	55-177	55-177	FP0, FP1
AAS	FPATAN		9-75	44-167	44-167	FP0, FP1
DAA	FSCALE		5	27		FP0, FP1
DAS	FEXTRACT		7	9	6	FP0, FP1
AAD	F2XM1		8	32-37		FP0, FP1
AAM	FYL2X		8-51	30-120	30-120	FP0, FP1
MUL, IMUL	FYL2XP1		61	~160	~160	FP0, FP1
MUL, IMUL	r16/m16	3	3	3	10	
MUL, IMUL	r32/m32	2	3	2	10	
MUL, IMUL	r64/m64	2	6	5	10	
IMUL	r16,r16/m16	1	3	1	10	
IMUL	r32,r32/m32	1	3	1	10	
IMUL	r64,r64/m64	1	6	4	10	
IMUL	r16,(r16),i	2	4	1	10	
IMUL	r32,(r32),i	1	3	1	10	
IMUL	r64,(r64),i	1	6	4	10	
DIV	r8/m8	1	11-14	11-14	10	
DIV	r16/m16	2	12-19	12-19	10	
DIV	r32/m32	2	12-27	12-27	10	
DIV	r64/m64	2	12-43	12-43	10	
IDIV	r8/m8	1	11-14	11-14	10	
IDIV	r16/m16	2	12-19	12-19	10	
IDIV	r32/m32	2	12-27	12-27	10	
IDIV	r64/m64	2	12-43	12-43	10	

Note: Two micro-operations can execute simultaneously if they go to different execution pipes



# Instruction Cost

```

ics
& (depth < MAXDEPTH)
c = inside ? 1 : 0;
nt = nt / nc; ddn = ddn / dnc;
os2t = 1.0f - nnt * ddn;
D, N );
0)
at a = nt - nc; b = nt - nc;
at Tr = 1 - (R0 + (1 - R0)
Tr) R = (D * nnt - N * (ddn
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, AlignedRadiance
e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following
ive)
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```

# Intel Skylake 2015

ADD SUB	r,r/i	1	1	p0156	1	0.25	
ADD SUB	r,m	1	2	p0156 p23		0.5	
ADD SUB	m,r/i	2	4	2p0156 2p237 p4	5	1	
ADC SBB	r,r/i	1	1	p06	1	1	
ADC SBB	r,m	2	2	p06 p23		1	
ADC SBB	m,r/i	4	6	3p0156 2p237 p4	5	2	
CMP	r,r/i	1	1	p0156	1	0.25	
CMP	m,r/i	1	2	p0156 p23	1	0.5	
INC DEC NEG	r	1	1	p0156	1	0.25	
NOT							
INC DEC NOT	m	3	4	p0156 2p237 p4	5-6	1	
NEG	m	2	4	p0156 2p237 p4	5-6	1	
AAA		2	2	p1 p56	4		not 64 bit
AAS		2	2	p1 p56	4		not 64 bit
DAA DAS		2	2	p1 p56	4		not 64 bit
AAD							
AAM							
MUL IMUL							
MUL IMUL							
MUL IMUL							
MUL IMUL							
MUL IMUL							
MUL IMUL							
MUL IMUL							
MUL IMUL							
IMUL							
IMUL	r,m	1	2	p1 p23	1		
IMUL	r16,r16,i	2	2	p1 p0156	4	1	
IMUL	r32,r32,i	1	1	p1	3	1	
IMUL	r64,r64,i	1	1	p1	3	1	
IMUL	r16,m16,i	2	3	p1 p0156 p23		1	
IMUL	r32,m32,i	1	2	p1 p23		1	
IMUL	r64,m64,i	1	2	p1 p23		1	
MULX	r32,r32,r32	3	3	p1 2p056	4	1	BMI2
MULX	r32,r32,m32	3	4	p1 2p056 p23		1	BMI2
MULX	r64,r64,r64	2	2	p1 p5	4	1	BMI2

Math						
FSCALE		27	27			130
FXTRACT		17	17			11
FSQRT		1	1		p0	14-21
FSIN		53-105				50-120
FCOS		53-105				50-130
FSINCOS		55-120				55-150
F2XM1		16-90				65-80
FYL2X		40-100				103
FYL2XP1		56				77
FPTAN		40-112				140-160
FPATAN		30-160				100-160





# Instruction Cost

```

ics
& (depth < MAXDEPTH)
c = inside ? 1 : 0;
nt = nt / nc; ddn = ddn * ddn;
os2t = 1.0f - nnt * nnt;
D, N );
0)
at a = nt - nc; b = nt * n;
at Tr = 1 - (R0 + (1 - R0)
Tr) R = (D * nnt - N * (dd
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, &light;
e.x + radiance.y + radiance.z) > 0) && (depth <
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following
ive)
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```

## AMD “Zen 4” 2022

Arithmetic instructions							
ADD, SUB	r,r	1	1	0.25			
ADD, SUB	r,i	1	1	0.25			
ADD, SUB	r,m	1		0.33			
ADD, SUB	m,r8/16	2	7-8	1			
ADD, SUB	m,r32/64	2	1	1			may mirror
ADC, SBB	r,r	1	1	1			
ADC, SBB	r,i	1	1	1			
ADC, SBB	r,m	1	1	1			
ADC, SBB	m,r8/16	2	8	1			
ADC, SBB	m,r32/64	2	1	1			may mirror
ADCX ADOX	r,r	1	1	1			ADX
CMP	FSQRT			1	25	10	
CMP	FLDPI, etc.			1		1	
CMP	FSIN			12-60	50-200		P0 P1
CMP	FCOS			18-60	60-150		P0 P1
CMP	FSINCOS			12-100	80-150		P0 P1
INC, DEC, I	FPTAN			10-60	60-120		P0 P1
INC, DEC, I	FPATAN			10-100	50-190		P0 P1
AAA, AAS	FSCALE			8	11	4	P0 P1
DAA	FEXTRACT			13	12	5	P0 P1
DAS	F2XM1			10-18	50-60		P0 P1
AAD	FYL2X			10-60	40-60		P0 P1
AAM	FYL2XP1			70	~170		P0 P1
MUL, IMUL	r8/m8	4	13	4			
MUL, IMUL	r16/m16	1	3	1			
MUL, IMUL	r32/m32	3	3	2			
MUL, IMUL	r64/m64	2	3	1			
IMUL	r,r	1	3	1			
IMUL	r,m	1		1			
IMUL	r16,r16 i	2	4	1			



# Instruction Cost

What is the ‘cost’ of a single instruction?

The cost of a single instruction depends on a number of factors:

- The arithmetic complexity (sqrt > add);
- Whether the operands are in register or memory;
- The size of the operand (16 / 64 bit is often slightly slower);
- Whether we need the answer immediately or not (latency);
- Whether we work on signed or unsigned integers (DIV/IDIV).

On top of that, certain instructions can be executed simultaneously.





# Today's Agenda:

- The Cost of a Line of Code
- CPU Architecture: Instruction Pipeline
- Data Types and Their Cost
- Rules of Engagement



# Pipeline

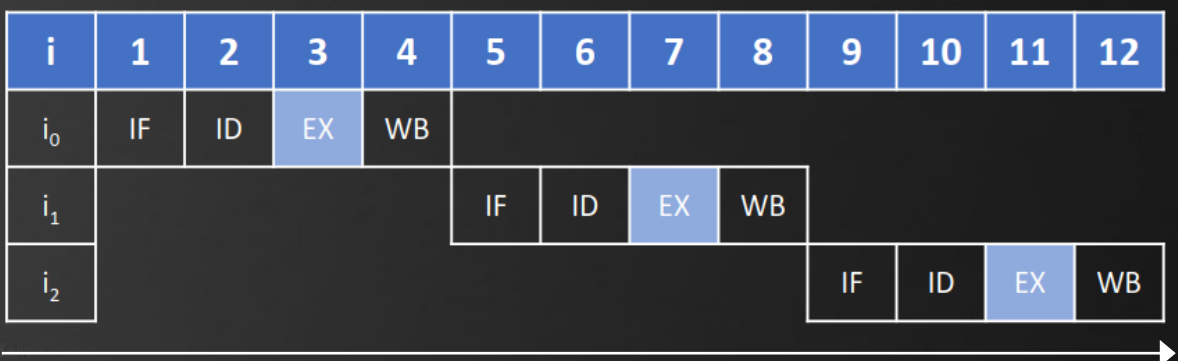
## CPU Instruction Pipeline

Instruction execution is typically divided in four phases:

1. Fetch            Get the instruction from RAM
2. Decode        Decode the byte code
3. Execute        Execute the instruction
4. Writeback     Write result to RAM/registers

```

ics
& (depth < MAXDEPTH)
c = inside ? 1.0f : 0.0f;
nt = nt / nc; ddn = ddn * nc;
ps2t = 1.0f - nnt * ddn;
D, N );
);
at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * c);
Tr) R = (D * nnt - N * (ddn *
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbab(ifi, diff
estimation doing
if;
radiance = SampleLight( &rand, I, &L, Albedo
2 * radiance.y - radiance.z) > 0;
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Pdf;
at3 factor = diffuse * pdf;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * rad
andom walk - done properly, closely following Surv
ive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2
urvive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```



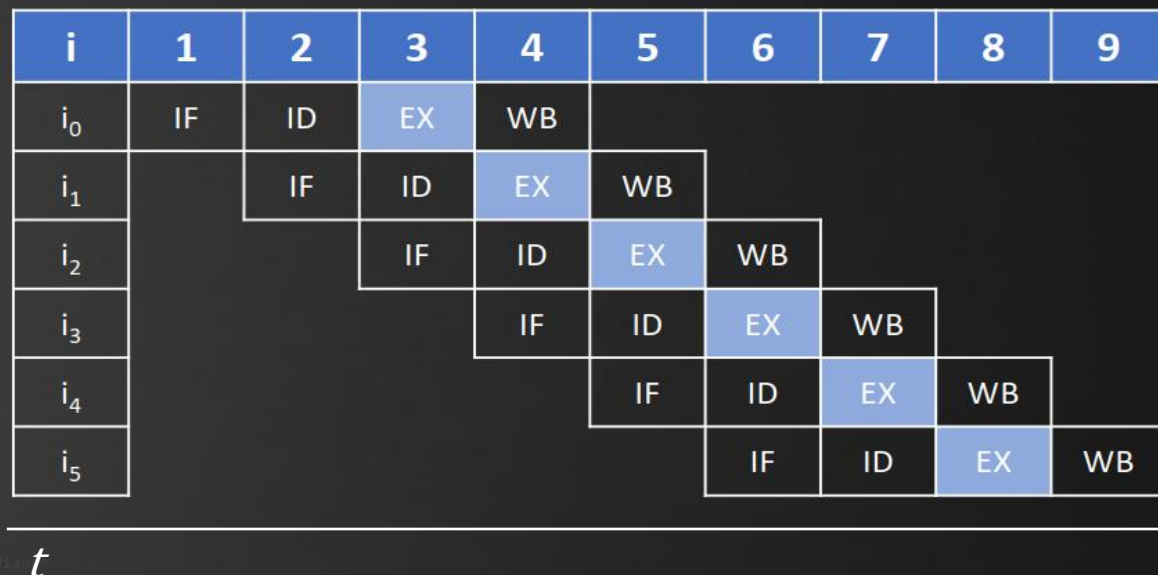
CPI = 4



# Pipeline

## CPU Instruction Pipeline

For each of the stages, different parts of the CPU are active. To use its transistors more efficiently, a modern processor overlaps these phases in a *pipeline*.



At the same clock speed, we get four times the throughput (CPI = IPC = 1).



# Pipeline

## CPU Instruction Pipeline

In practice, each of the pipeline phases takes several cycles to complete.

i	1	2	3	4	5	6	7	8	9
$i_0$	IF	ID	ID	ID	EX	WB			
$i_1$		IF			ID	ID	ID	EX	WB



```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn / nc;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
}

at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * nnt);
Tr) R = (D * nnt - N * (ddn * nnt));

E * diffuse;
= true;

efl + refr) && (depth < MAXDEPTH)
{
    D, N );
    refl * E * diffuse;
    = true;
}

MAXDEPTH)
{
    survive = SurvivalProbability( diffuse, i );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &align, &N );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
}

random walk - done properly, closely following
ive)
{
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
    
```



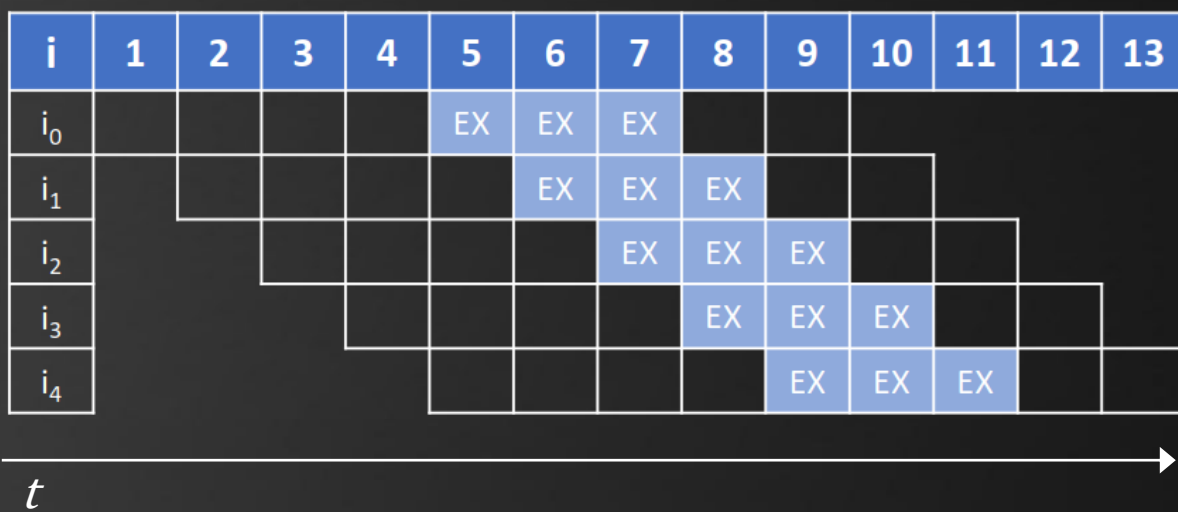
# Pipeline

## CPU Instruction Pipeline

Maximum clock speed is determined by the most complex of the four stages. For higher clock speeds, it is advantageous to increase the number of stages (thereby reducing the complexity of each individual stage).

```

ics
& (depth < MAXDEPTH)
c = inside ? 1.0f : 0.0f;
nt = nt / nc; dd = dd / nd;
ps2t = 1.0f - nnt;
D, N );
0);
at a = nt - nc, b = nt - nd;
at Tr = 1 - (R0 + (1 - R0) *
Tr) R = (D * nnt - N * (dd
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse,
estimation - doing it properly, closely followi
if;
radiance = SampleLight( &rand, I, &L, &light
e.x + radiance.y + radiance.z) > 0) && (rand
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely followi
ive)
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pd
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```



### Stages

- 7 PowerPC G4e
- 8 Cortex-A9
- 10 Athlon
- 12 Pentium Pro/II/III, Athlon 64
- 14 Core 2, Apple A7/A8
- 14/19 Core i2/i3 Sandy Bridge
- 16 PowerPC G5, Core i\*1 Nehalem
- 18 Bulldozer, Steamroller
- 20 Pentium 4
- 31 Pentium 4E Prescott

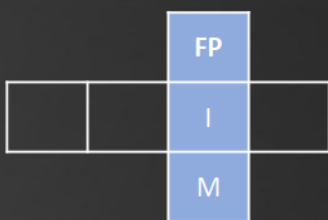
Super-pipelining allows higher clock speeds and thus higher throughput, but it also increases the latency of individual instructions.



# Pipeline

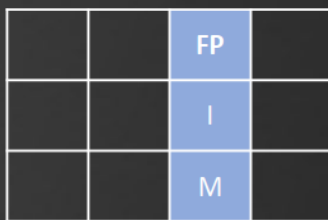
## CPU Instruction Pipeline

Different execution units for different (classes of) instructions:



Here, one execution unit handles floats;  
one handles integer;  
one handles memory operations.

Since the execution logic is typically the most complex part, we might just as well duplicate the other parts:





# Pipeline

## CPU Instruction Pipeline

This leads to the *superscalar* processor, which can execute multiple instructions in the same clock cycle, assuming not all instructions require the same execution logic.

i	1	2	3	4	5	6
fp <sub>0</sub>			EX			
int <sub>0</sub>			EX			
m <sub>0</sub>			EX			
fp <sub>1</sub>				EX		
int <sub>1</sub>				EX		
m <sub>1</sub>				EX		
fp <sub>2</sub>					EX	
int <sub>2</sub>					EX	
m <sub>2</sub>					EX	



IPC = 3 (or: ILP = 3)



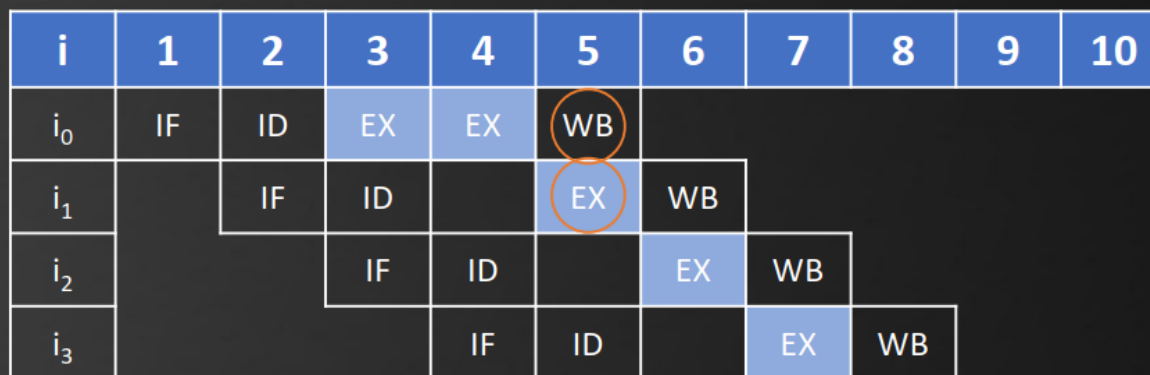
# Pipeline

## CPU Instruction Pipeline

Using a pipeline has consequences. Consider the following situation:

```

a = b * c;
d = a + 1;
y = y >> 1;
z = 0x1a4;
    
```



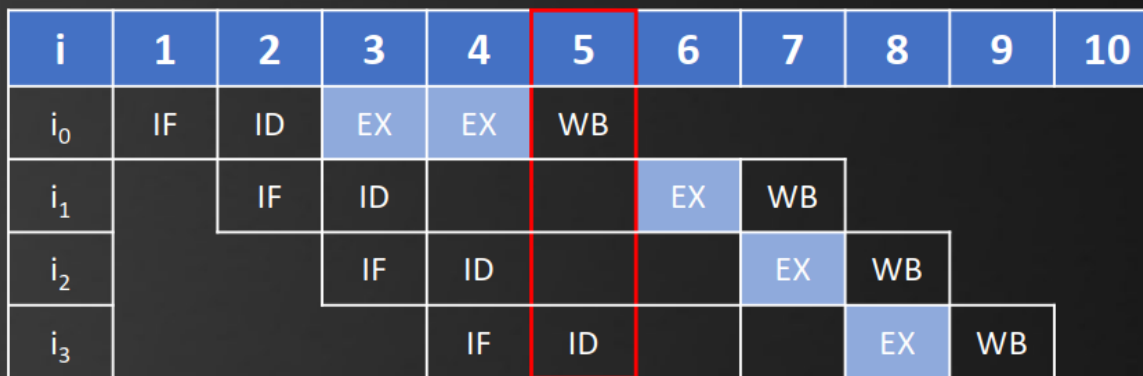
# Pipeline

## CPU Instruction Pipeline

Using a pipeline has consequences. Consider the following situation:

```

a = b * c;
d = a + 1;
y = y >> 1;
z = 0x1a4;
    
```



We now lose one cycle!



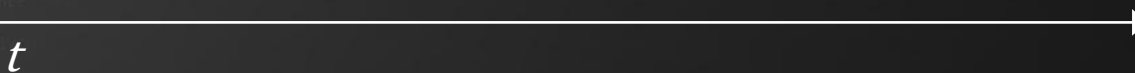
# Pipeline

## CPU Instruction Pipeline

Using a pipeline has consequences. Consider the following situation:

```
a = b * c;
d = a + 1;
y = y >> 1;
z = 0x1a4;
```

i	1	2	3	4	5	6	7	8	9	10
$i_0$	IF	ID	EX	EX	WB					
$i_1$		IF	ID			EX	WB			
$i_2$			IF	ID	EX			WB		
$i_3$				IF	ID		EX			WB



Out-of-order execution requires instructions to be *retired* in-order. An instruction is retired when its result is written back to memory.



# Pipeline

## CPU Instruction Pipeline

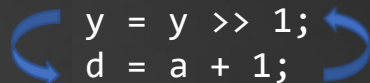
A good compiler re-organizes your code to maximize throughput.

```
a = b * c;
```

```
y = y >> 1;
```

```
d = a + 1;
```

```
z = 0x1a4;
```



i	1	2	3	4	5	6	7	8	9	10
$i_0$	IF	ID	EX	EX	WB					
$i_1$		IF	ID		EX	WB				
$i_2$			IF	ID		EX	WB			
$i_3$				IF	ID		EX	WB		



# Pipeline

## CPU Instruction Pipeline

So how do we utilize out-of-order execution:

- A compiler reorganizes code to prevent latencies
- The CPU reorganizes instructions to prevent latencies
- Feeding mixed code provides compiler and CPU with opportunities for shuffling

One little problem remains:

*What if the CPU doesn't know what the next instruction is?*





# Pipeline

## Speculative Execution

The CPU *guesses* what a branch will do:

```
if (a < b) foo(); else bar();
```

For this it uses *branch prediction*.

Instructions at the predicted location will be executed even if it is not sure that this is correct.

A *branch misprediction* requires that the pipeline is flushed and re-populated...

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn / nc;
        ps2t = 1.0f + nnt * nnt;
        D, N );
    }
}

at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * R);
Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
}

efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
}

MAXDEPTH)
survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, &light;
e.x + radiance.y + radiance.z) > 0) && (rand
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following
ive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;

```



# Today's Agenda:

- The Cost of a Line of Code
- CPU Architecture: Instruction Pipeline
- Data Types and Their Cost
- Rules of Engagement

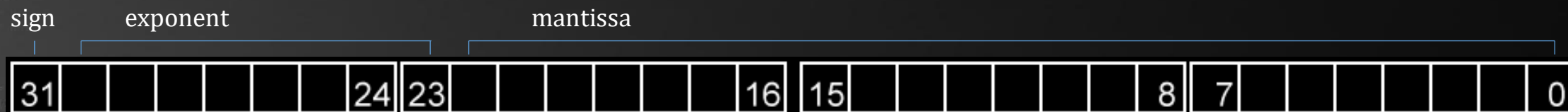




# Data Types

## Data types in C++

### float



Size: 32 bit (4 bytes)

Exponent: 8 bit; -127 ... 128

Mantissa: 23 bit; 0 ...  $2^{23} - 1$

Value:  $\text{sign} * \text{mantissa} * 2^{\text{exponent}}$

Exercise: write a function that replaces array  $a = \{ 0.5, 0.25, 0.125, 0.0625, \dots \}$ .



# Data Types

## Data types in C++

double	64 bit (8 bytes)
char, unsigned char	8 bit
short, unsigned short	16 bit
LONG	32 bit (same as int)
LONG LONG, __int64	64 bit
bool	8 bit (!)

## Padding\*:

```

struct Test
{
    unsigned int u;
    bool flag;
};
// sizeof( Test ) is 8
    
```

```

struct Test2
{
    double d;
    bool flag;
};
// sizeof( Test2 ) is 16
    
```

\*More on <http://www.catb.org/esr/structure-packing>



# Data Types

## Data types in C++ - Conversions

### Explicit:

```
float fpi = 3.141593;
int pi = (int)(1024.0f * fpi);
```

### Implicit:

```
struct Color { unsigned char a, r, g, b; };
Color bitmap[640 * 480];
for( int i = 0; i < 640 * 480; i++ )
{
    bitmap[i].r *= 0.5f;
    bitmap[i].g *= 0.5f;
    bitmap[i].b *= 0.5f;
}
```

```
// bitmap[i].r *= 0.5f;
movzx    eax,byte ptr [ecx-1]
mov      dword ptr [ebp-4],eax
fild     dword ptr [ebp-4]
fstcw    word ptr [ebp-2]
movzx    eax,word ptr [ebp-2]
or       eax,0C00h
mov      dword ptr [ebp-8],eax
fmul     st,st(1)
fldcw    word ptr [ebp-8]
fistp    dword ptr [ebp-8]
movzx    eax,byte ptr [ebp-8]
mov      byte ptr [ecx-1],al
```





# Data Types

## Data types in C++ - Conversions

### Explicit:

```
float fpi = 3.141593;
int pi = (int)(1024.0f * fpi);
```

### Avoiding conversion:

```
struct Color { unsigned char a, r, g, b; };
Color bitmap[640 * 480];
for( int i = 0; i < 640 * 480; i++ )
```

```
    bitmap[i].r >>= 1;
    bitmap[i].g >>= 1;
    bitmap[i].b >>= 1;
```

```
// bitmap[i].r >>= 1;
shr          byte ptr [eax-1],1
// bitmap[i].g >>= 1;
shr          byte ptr [eax],1
// bitmap[i].b >>= 1;
shr          byte ptr [eax+1],1
```



# Data Types

## Data types in C++ - Conversions

Explicit:

```
float fpi = 3.141593;
int pi = (int)(1024.0f * fpi);
```

Avoiding conversion (2):

```
struct Color { union { struct { unsigned char a, r, g, b; }; int argb; }; };
Color bitmap[640 * 480];
for( int i = 0; i < 640 * 480; i++ )
    bitmap[i].argb = (bitmap[i].argb >> 1) & 0x7f7f7f;
```



# Today's Agenda:

- The Cost of a Line of Code
- CPU Architecture: Instruction Pipeline
- Data Types and Their Cost
- Rules of Engagement



# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 1: Avoid Costly Operations

- Replace multiplications by bitshifts, when possible
- Replace divisions by (reciprocal) multiplications
- Avoid sin, cos, sqrt

```

ics
& (depth < MAXDEPTH)
{
    if (inside & !isRefr)
    {
        nt = nt / nc; ddn = sqrt(1 - nt * nt);
        r = 1.0f - nnt * nnt;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * r);
    R = (D * nnt - N * (ddn * r +
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse, r);
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &light);
    e.x + radiance.y + radiance.z) > 0) && (abs(radiance.x) +
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf);
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```



# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 2: Precalculate

- Reuse (partial) results
- Adapt previous results (interpolation, reprojection, ...)
- Loop hoisting
- Lookup tables

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = abs(ddn);
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * ddn);
    (Tr) R = (D * nnt - N * (ddn > 0 ? 1 : -1));
    E * diffuse;
    = true;
    -
    refl + refr) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &light );
    e.x + radiance.y + radiance.z) > 0) && (abs(radiance.x) +
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
  }
}

```





# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 4: Avoid Conditional Branches

- if, while, ?, MIN/MAX
- Try to split loops with conditional paths into multiple unconditional loops
- Use lookup tables to prevent conditional code
- Use loop unrolling
- If all else fails: make conditional branches predictable

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
}

at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * r);
Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
}

efl + refr)) && (depth < MAXDEPTH)
{
    D, N );
    refl * E * diffuse;
    = true;
}

MAXDEPTH)
{
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &light);
    e.x + radiance.y + radiance.z) > 0) && (depth <
    v = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
}

random walk - done properly, closely following
ive)
{
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf);
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}

```





# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 5: Early Out

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (Dd
    E * diffuse;
    = true;
    refl + refr) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse, r
    estimation - doing it properly, closely
    if;
    radiance = SampleLight( &rand, I, N, A, Aligned
    e.x + radiance.y + radiance.z) > 0) && (rand
    v = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following S
    (vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, Spdf
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```

**char** a[] = “abcdefghijklmnpqrstuvwxyz”;  
**char** c = ‘p’;  
**int** position = -1;  
**for** ( **int** t = 0; t < strlen( a ); t++ )
 {
 **if** (a[t] == c)
 {
 position = t;
 }
 }

```

char a[] = “abcdefghijklmnpqrstuvwxyz”;  

char c = ‘p’;  

int position = -1, len = strlen( a );  

for ( int t = 0; t < len; t++ )
    {
        if (a[t] == c)
        {
            position = t;  

            break;  

        }
    }

```



# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 6: Use the Power of Two

- A multiplication / division by a power of two is a (cheap) bitshift
- A 2D array lookup is a multiplication too – make ‘width’ a power of 2
- Dividing a circle in 256 or 512 works just as well as 360 (but it’s faster)
- Bitmasking (for free modulo) requires powers of 2

1-2-4-8-16-32-64-128-256-512-1024-2048-4096-8192-16384-32768-65536

Be fluent with powers of 2 (up to  $2^{16}$ );  
 learn to go back and forth for these:  $2^9 = 512 = 2^9$ .  
 Practice counting from 0..31 on one hand in binary.

```

ics
& (depth < MAXDEPTH)
c = inside ? 1.0f : 0.0f;
nt = nt / nc; ddn = ddn * ddn;
os2t = 1.0f - nnt * nnt;
D, N );
);
at a = nt - nc; b = nt * nc;
at Tr = 1 - (R0 + (1 - R0) * c);
Tr) R = (D * nnt - N * (ddn *
E * diffuse;
= true;
efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse, r1, r2, &R, Spdf );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, Align );
e.x + radiance.y + radiance.z) > 0) && (survive)
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following
ive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, Spdf );
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
    
```



# Rules of Engagement

## Common Opportunities in Low-level Optimization

### RULE 7: Do Things Simultaneously

- Use those cores
- An integer holds four bytes; use these for instruction level parallelism
- More on this later.

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }

    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;

    refl + refr) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;

    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &align,
    e.x + radiance.y + radiance.z) > 0) && (abs(
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following
    survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```





# Today's Agenda:

- The Cost of a Line of Code
- CPU Architecture: Instruction Pipeline
- Data Types and Their Cost
- Rules of Engagement



/INFOMOV/

END of “Low Level”

next lecture: “Profiling”

```
ics
& (depth < MAXDEPTH)
{
    if ( ! inside )
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
}

at a = nt - nc; b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * r);
Tr) R = (D * nnt - N * (ddn * nnt));

E * diffuse;
= true;

efl + refr) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;

MAXDEPTH)

survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, &align,
e.x + radiance.y + radiance.z) > 0) && (abs(radiance.x
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance

random walk - done properly, closely following
ive)

;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
urvive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```

