PROBLEMS

6.1. Suppose that a man's job is to install 98 rivets in the right wing of an airplane under construction. If the random variable T is the total time required for one airplane, then what is the approximate distribution of T?

6.2. Prove comment 2 for the Weibull distribution in Table 6.3.

6.3. Prove comment 2 for the Pearson type VI distribution in Table 6.3.

6.4. Consider a four-parameter Pearson type VI distribution with shape parameters α_1 and α_2, scale parameter β, and location parameter γ. If $\alpha_1 = 1$, $\gamma = \beta = c > 0$, then the resulting density is

$$f(x) = \alpha_2 x^{-(\alpha_2+1)} c^{\alpha_2} \qquad \text{for } x > c$$

which is the density function of a *Pareto distribution* with parameters c and α_2, denoted Pareto(c, α_2). Show that $X \sim$ Pareto(c, α_2) if and only if $Y = \ln X \sim$ expo($\ln c$, $1/\alpha_2$), an exponential distribution with location parameter $\ln c$ and scale parameter $1/\alpha_2$.

6.5. For the empirical distribution given by $F(x)$ in Sec. 6.2.4, discuss the merit of defining $F(X_{(i)}) = i/n$ for $i = 1, 2, \ldots, n$, which seems like an intuitive definition. In this case, how would you define $F(x)$ for $0 \le x < X_{(1)}$?

6.6. Compute the expectation of the empirical distribution given by $F(x)$ in Sec. 6.2.4.

6.7. For discrete distributions, prove that the histogram (Sec. 6.4.2) is an unbiased estimator of the (unknown) mass function; i.e., show that $E(h_j) = p(x_j)$ for all j. *Hint:* For j fixed, define

$$Y_i = \begin{cases} 1 & \text{if } X_i = x_j \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = 1, 2, \ldots, n$$

6.8. Suppose that the histogram of your observed data has several local modes (see Fig. 6.31), but that it is not possible to break the data into natural groups with a different probability distribution fitting each group. Describe an alternative approach for modeling your data.

6.9. For a geometric distribution with parameter p, explain why the MLE $\hat{p} = 1/[\bar{X}(n) + 1]$ is intuitive.

6.10. For each of the following distributions, derive formulas for the MLEs of the indicated parameters. Assume that we have IID data $X_1, X_2, \ldots, X_n$ from the distribution in question.

(*a*) U(0, b), MLE for b
(*b*) U(a, 0), MLE for a
(*c*) U(a, b), joint MLEs for a and b
(*d*) N(μ, σ^2), joint MLEs for μ and σ
(*e*) LN(μ, σ^2), joint MLEs for μ and σ
(*f*) Bernoulli(p), MLE for p
(*g*) DU(i, j), joint MLEs for i and j