

Simulation

Exercise 1.26

System description, input data/distributions:
see description of the exercise

Assumptions:
no new assumptions are made

State:
server A_1 idle/busy1 (i.e. busy with type 1 customer) /busy2
server A_2 idle/busy1/busy2
server B idle/busy1/busy2
number of customers in queue 1 with their arrival times
number of customers in queue 2 with their arrival times.

Events:
arrival of customer
departure1A: departure of type 1 customer from server of type A
departure1B: departure of type 1 customer from server of type B
departure of type 2 customer

Performance measures:
average delay in queue for each type of customer:
Let D_1 be the total delay of type 1 customers in the simulation and n_1 be the number of type 1 customers for which delay has been measured. If a customer enters service, increase D_1 by the delay of this customer and increase n_1 by 1. At the end of the simulation the average delay equals D_1/n_1 . Same is applied for customers of type 2.
average number of customers in queue 1 and queue 2.
Note that this is an average over time. Let Q_1 be the surface under the graph of the queue-length of queue 1 as a function of time until the current time. At each event we update Q_1 by adding $(time - time_{previous\ event}) * (queue\ length\ during\ this\ interval)$. At the end the average number of customers in the queue equals Q_1 divided by the total time. Similarly, we have Q_2 .
expected portion of time that each server spends on customer of type 1 and type 2
Let $T_{1,S}$ be the amount of time that server S ($S=A_1, A_2, B$) is busy until the current time. At each event update $T_{1,S}$ by adding $time - time_{previous\ event}$ if the server S was busy working for customer 1 during the interval $[time_{previous\ event}, time]$. Similarly, we have the number $T_{2,S}$. At the end, these numbers have to be divided by the total time.

Pseudo-code:

```
while time < runlength
{
case nextevent of
  arrival:
  {
    schedule new arrival;
    determine client type (with random generator);
    update busy time of servers and total queue lengths;
    if type 1 customer
      if (all servers busy) add customer to queue
      else{ update  $D_1$ ;
            if server of type A is available {
              start service on type A server;
              schedule departure type 1 from type A server;
            }
            else{ start service on type B server;
                  schedule departure type 1 from type B server;
                }
          }
  }
}
```

```

    }
else /* type 2 customer */
    if (server A and B available){
        update  $D_2$ ;
        start service;
        schedule departure2;
    }
    else add customer to queue
}
}
departure1A:
{
    update busy time of servers and total queue lengths;
    set server idle;
    if (server A and B are available and queue2 not empty)
    {
        update  $D_2$ ;
        start service type 2 customer;
        schedule departure2;
    }
    else
        if queue1 is not empty
        {
            update  $D_1$ ;
            start service type 1 customer;
            schedule new departure1A
        }
}
departure 1B:
    similar
departure2: /* almost the same */
{
    update busy time of servers and total queue lengths,
    set one more server A and set server B idle
    if (queue2 not empty)
    {
        update  $D_2$ ;
        start service type 2 customer;
        schedule new departure2
    }
    else
        if queue1 is not empty
        {
            update  $D_1$ ;
            start service type 1 customer on server A;
            schedule departure1A;
            if queue1 is still not empty
            {
                update  $D_1$ ;
                start service type 1 customer onserverB;
                schedule departure1B
            }
        }
}
}
}

```

NB: To determine the amount of time that each of the servers spends on customers of type 1 and 2, we also need to know for each departure from a server A, from which of the two servers this departure takes place. This detail is not included in the pseudo-code.